

Influence Analysis of Displacement Currents in Geoelectromagnetic Methods

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Abstract

The high frequency electromagnetic method and transient electromagnetic method are widely used recently. However, the influence of displacement currents is significant and can't be ignored for high frequencies case in frequency domain and in extremely early time for time domain method. So research on the influence of displacement current effect is of great importance. The comparisons of considering and ignoring displacement current effect were carried out both in frequency and time domains. The half-space and layered homogeneous models were adopted in simulations. The results show that the displacement current must be taken into account for high frequency, great resistivity and dielectric constant problems in frequency domain. And for the time domain case, the displacement current effect is mainly reflected in the earlier time response.

Keywords: Displacement Current Effect, Magnetic Dipole, Green Function, Frequency Domain, Time Domain.

1. Introduction

For most real geo-exploration problems, the displacement currents are neglected which is mainly due to the relative low frequency of sources and low resistivity of environments. However, for high frequencies and high resistivity, such as permafrost, magmatic rocks, dry sands, carbonates, etc, the influence of displacement currents is significant^[1]. And also, the displacement current effect is dominant in extremely early time for time domain method.

In the frequency domain shallow sounding applications, the minimum frequency is about 100 kHz. And for more shallower structure, the higher frequency even to 100k ~ 100MHz electromagnetic (EM) method^[2] is required. Sinha^[3] have proved that the influence of displacement current might be significant at frequencies greater than roughly 100 kHz in very high resistivity environments. Zheng^[4-5] concluded that for most medium, the impact of displacement current must be considered when the frequency is greater than 100 kHz, and the effect increases as frequency increasing.

For the time-domain EM method as transient electromagnetic (TEM) method, the shallow layer information is mainly reflected in the early time response^[6]. With the development of devices and equipment, the turn off-time for emission current has been shortened from the

millisecond to microsecond, even to nanoseconds level. And the corresponding frequencies are raise to 10MHz or 100MHz. The displacement current is crucial for earlier response.

Therefore, research on the displacement current is of great importance. The comparisons of considering and ignoring displacement current were carried out both in frequency and time domain. In section 2, the frequency domain magnetic field amplitude responses versus frequency, dielectric constant and resistivity were compared in half-space and layered homogeneous model. And in section 3, the whole space scalar Green functions for time domain were compared theoretically. The simulations were carried out in section 4.

2. Influence analysis in frequency domain

For homogeneous half-space and layered model problems, the source of vertical magnetic dipole is adopted. The difference between considering and ignoring the displacement current mainly reflects in the wave number calculation. So we firstly derived the expressions of magnetic-field component for neglecting the displacement current, and then modify the wave number formula to acquire the solution that takes the displacement current into account.

2.1 The magnetic-field of vertical magnetic dipole in homogeneous half-space

For the magnetic dipole, we have divergence equation $\nabla \cdot \mathbf{D} = 0$. So the following formulas are obtained according to the Maxwell equations and by substituting $\mathbf{E} = -i\omega\mu\nabla \times \mathbf{A}^*$, $k^2 = -i\omega\mu\sigma$

$$\begin{cases} \nabla^2 \mathbf{A}^* + k^2 \mathbf{A}^* = 0 \\ \mathbf{E} = -i\omega\mu\nabla \times \mathbf{A}^* \\ \mathbf{H} = k^2 \mathbf{A}^* + \nabla (\nabla \cdot \mathbf{A}^*) \end{cases} \quad (1)$$

In particularly, the magnetic dipole source is located at the interface. The distance formula $R = \sqrt{\rho^2 + z^2}$ and $\partial R/\partial z = zR^{-1}$ are satisfied. So the magnetic field component of z -axis direction on the interface is

$$H_z = \frac{M}{2\pi} \int_0^\infty \frac{m^3}{m + m_1} J_0(\rho m) dm, z = 0 \tag{2}$$

Where $m_i = \sqrt{m^2 - k_{i1}^2}$, $i=0,1$ which means ignoring or considering the displacement current. For example, when ignoring displacement current, we have $i=0$ and the wave number $k_{i0}^2 = -i\omega\mu\sigma$. However, for considering case, we have $i=1$ and $k_{i1}^2 = \omega^2\mu\varepsilon - i\omega\mu\sigma$. In order to calculate equation (2), the expressions can be modified as

$$H_z = \frac{M}{2\pi k_1^2} \int_0^\infty (m^4 - m^3 m_1) J_0(\rho m) dm = \frac{M}{2\pi k_1^2} \int_0^\infty [m^4 - (m_1^2 + k_1^2) m m_1] J_0(\rho m) dm \tag{3}$$

By using Sommerfeld integral identity

$$\frac{e^{-ik_j R}}{R} = \int_0^\infty \frac{m}{m_j} e^{-m_j |z|} J_0(\rho m) dm \tag{4}$$

We obtain

$$\int_0^\infty m m_j J_0(\rho m) dm = \left. \frac{\partial^2}{\partial z^2} \left(\frac{e^{-ik_j R}}{R} \right) \right|_{z=0} = (-\rho^{-3} - ik_j \rho^{-2}) e^{-ik_j R} \tag{5}$$

$$\int_0^\infty m m_j^3 J_0(\rho m) dm = \left. \frac{\partial^4}{\partial z^4} \left(\frac{e^{-ik_j R}}{R} \right) \right|_{z=0} = (9\rho^{-5} + 9ik_j \rho^{-4} - 3k_j^2 \rho^{-3}) e^{-ik_j R} \tag{6}$$

Because the magnetic dipole source is located at the interface, $k_1=0$ and $m_1=m$. Then

$$\int_0^\infty m^4 J_0(\rho m) dm = 9\rho^{-5} \tag{7}$$

Substituting the equations (5) and (6) into equation (2), the following formula is obtained

$$H_z = \frac{9M}{2\pi k_{i1}^2 \rho^5} \left[1 - \left(1 + ik_{i1} \rho - \frac{4}{9} k_{i1}^2 \rho^2 - \frac{1}{9} ik_{i1}^3 \rho^3 \right) e^{-ik_{i1} \rho} \right] \tag{8}$$

2.2 The magnetic-field of vertical magnetic dipole in layered model

In the same way, the magnetic dipole source is located at the interface for layered model. The Hankel transforms expressions of H_z is

$$H_z = \frac{M}{2\pi} \int_0^\infty \frac{m^3}{m + m_1/R_1^*} J_0(\rho m) dm \tag{9}$$

Where $m_j = \sqrt{m^2 - k_{ji}^2}$, and $j=1, 2, \dots, n$. which represents the layer's number, $i=0,1$ is defined as before. The thickness and resistivity of every layer are respectively represented as $(h_1, \rho_1), \dots, (h_{n-1}, \rho_{n-1})$ and $(h_n \rightarrow \infty, \rho_n)$. Taking $th(m_1 h_1) = \frac{1 - e^{-2m_1 h_1}}{1 + e^{-2m_1 h_1}}$ and R_1^* can be obtained by recursive formula as

$$\begin{cases} R_n^* = 1 \\ R_j^* = \frac{m_j R_{j+1}^* + m_{j+1} th(m_j h_j)}{m_{j+1} + m_j R_{j+1}^* th(m_j h_j)}, j = 1, 2, \dots, n-1 \end{cases} \tag{10}$$

In the numerical calculation the Hankel transform equation (9), the digital filtering method can be adopted. For a v th-order Hankel transforms

$$g(\gamma) = \int_0^\infty \lambda f(\lambda) J_\nu(\lambda \gamma) d\lambda, \nu > -1 \tag{11}$$

Where $J_\nu(x)$ is the ν -order Bessel function of the first kind, $g(\gamma)$ is transformed function of input function $f(\lambda)$. The numerical calculation formula of filtering method^[7-8] is

$$g(\gamma) = \frac{1}{r} \sum_{i=1}^n \lambda_i f(\lambda_i) C_i \tag{12}$$

Choosing the sampling nodes $\lambda_i = (1/r) \times 10^{a+(i-1)s}$ for kernel function $\lambda f(\lambda)$, C_i is the filter coefficient. The exact parameters as a, s and C_i can be found in filtering method for different nodes schemes.

3. Influence analysis in time domain

The direct response of source in free space has practical significance in geophysical applications. Because the Green function is defined as the response of unit source which placed at the origin of coordinate. In this section, the whole space time domain Green function was compared both in considering and ignoring the displacement current effect.

3.1 Considering the displacement current

The Green function caused by the unit source in whole space is

$$G(r) = \frac{e^{-ikr}}{4\pi r} \quad (13)$$

Taking $s = i\omega$ and then

$$ik = \sqrt{-k^2} = (\mu\epsilon)^{1/2} (s^2 + s\sigma/\epsilon)^{1/2} \quad (14)$$

So the Green function in complex frequency domain is in general form of

$$G(r, s) = \frac{e^{-(\mu\epsilon)^{1/2}(s^2+s\sigma/\epsilon)^{1/2}r}}{4\pi r} = \frac{e^{-\alpha(s^2+s\sigma/\epsilon)^{1/2}}}{4\pi r} \quad (15)$$

$$= -\frac{1}{4\pi r} \frac{\partial}{\partial \alpha} \left[\frac{e^{-\alpha(s^2+s\sigma/\epsilon)^{1/2}}}{(s^2+s\sigma/\epsilon)^{1/2}} \right]$$

Where $\alpha=(\mu\epsilon)^{1/2}r$, then using the following Laplace transform formula

$$\frac{e^{-\alpha\sqrt{(s+\sigma/2\epsilon)^2-(\sigma/2\epsilon)^2}}}{\sqrt{(s+\sigma/2\epsilon)^2-(\sigma/2\epsilon)^2}} : \quad (16)$$

$$e^{-(\sigma/2\epsilon)t} I_0(\sigma/2\epsilon\sqrt{t^2-\alpha^2}) u(t-\alpha)$$

So we can obtain the expression of the Green function in time domain as

$$G(r, t) = -\frac{1}{4\pi r} \frac{\partial}{\partial \alpha} \left\{ e^{-\frac{\sigma}{2\epsilon}t} I_0 \left[\frac{\sigma}{2\epsilon} \sqrt{t^2-\alpha^2} \right] u(t-\alpha) \right\}$$

$$= \frac{1}{4\pi r} \left\{ \frac{e^{-\frac{\sigma}{2\epsilon}t} \left(\frac{\sigma}{2\epsilon} \right) \alpha}{\sqrt{t^2-\alpha^2}} I_1 \left[\frac{\sigma}{2\epsilon} \sqrt{t^2-\alpha^2} \right] u(t-\alpha) + \delta(t-\alpha) e^{-\frac{\sigma}{2\epsilon}\alpha} \right\} \quad (17)$$

Let $a = \sigma/2\epsilon$, $\tau_0 = \alpha = \sqrt{\mu\epsilon}r$ and using the equation $I'_o(x) = I_1(x)$, we can obtain

$$G(r, t) = \frac{1}{4\pi r} \left\{ e^{-at} \frac{a\tau_0}{\sqrt{t^2-\tau_0^2}} I_1 \left[a\sqrt{t^2-\tau_0^2} \right] u(t-\tau_0) + \delta(t-\tau_0) e^{-a\tau_0} \right\} \quad (18)$$

3.2 Ignoring the displacement current

When ignoring the displacement current effect, the Green function in complex frequency domain is

$$G(r, s) = \frac{e^{-(\mu\sigma s)^{1/2}r}}{4\pi r} \quad (19)$$

According to the Laplace transform table and using the following pair

$$e^{-\alpha s^{1/2}} : \frac{\alpha}{2\pi^{1/2}t^{3/2}} e^{-\alpha^2/4t} \quad (20)$$

Taking $\alpha=(\mu\epsilon)^{1/2}r$, then the Green function in time domain is

$$G(r, t) = \frac{(\mu\sigma)^{1/2}}{8\pi^{3/2}t^{3/2}} e^{-\mu\sigma r^2/4t} u(t) \quad (21)$$

4. Simulation analysis

4.1 Simulation of magnetic-field in homogeneous half-space

Considering the high frequency EM method application, the frequency range of vertical magnetic dipole is 100k-100MHz. Setting the magnetic moment $M=1A\cdot m^2$, the permeability $\mu=\mu_0=4\pi\times 10^{-7}H/m$. Calculating the magnetic-field amplitude response which is located in 5m distance away from magnetic dipole. The following two case of varied dielectric constant or resistivity were investigated, the amplitude response ratio curves of considering and ignoring displacement current was illustrated.

1) Case 1. Setting the relative dielectric constant $\epsilon_r=10$, the resistivity is varied from 20Ω-m, 100Ω-m, 500Ω-m to 1000Ω-m. The ratio curves were shown in figure 1.

2) Case 2. Setting the resistivity $\rho=300\Omega\cdot m$, the relative dielectric constant is varied from 1, 5, 10, 20 to 30. The ratio curves were shown in figure 2.

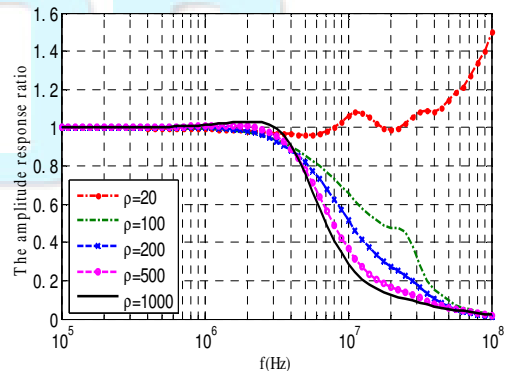


Fig.1 Ratio curves versus frequency for constant dielectric

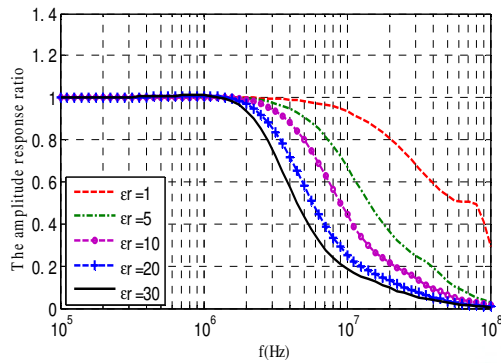


Fig.2 Ratio curves versus frequency for constant dielectric

It can be seen from figure 1 that the amplitude response ratio is stable and close to 1 when the frequency lower than 3MHz which means the displacement current effect can be ignored and the displacement current is much smaller than the conduction current. However, when increasing the source frequency, the magnetic field for ignoring displacement current is significantly less than the one that considering the displacement current. And at the same time, when resistivity increasing, the amplitude response ratio becomes small which means the displacement current plays much more important role for high resistivity case.

From figure 2, we can conclude that the amplitude responses differ small when the frequency is lower than 2MHz. The displacement current effects become more apparent as the frequency increasing, and the larger dielectric constant yield the smaller amplitude response ratio. Therefore, when increasing the frequency to a certain degree, the displacement current effect must be taken into account in homogeneous half-space.

4.2 Simulation of magnetic-field in layered model

Considering a two layered model, setting the relative dielectric constant $\epsilon_r=10$, the magnetic moment $M=1A \cdot m^2$, and the permeability $\mu=\mu_0=4\pi \times 10^{-7}H/m$. The thickness of the first layer is $h_1=10m$. The magnetic-field amplitude response which is located in 5m distance away from magnetic dipole was calculated and compared. And two-type of model structure were carried out. The first is a low resistivity covering case where $\rho_1=10^2\Omega \cdot m$, $\rho_2=10^4\Omega \cdot m$ and the second is a high resistivity covering as $\rho_1=10^4\Omega \cdot m$, $\rho_2=10^2\Omega \cdot m$. Then the ratio curves were shown in figure 3~4. In calculation the H_z by equation (9), the Hankel transform was implemented by the filtering method with 801 sampling nodes scheme.

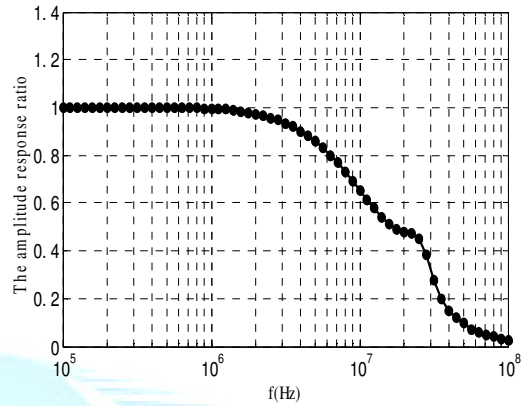


Fig.3 $\rho_1=10^2\Omega \cdot m$, $\rho_2=10^4$

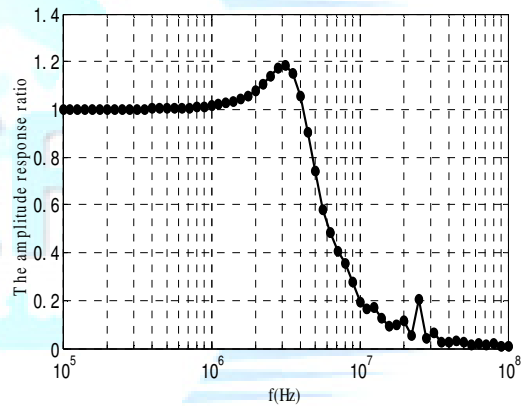


Fig.4 $\rho_1=10^4\Omega \cdot m$, $\rho_2=10^2\Omega \cdot m$

Obviously, no matter the resistivity distribution is a low resistivity covering (Fig.3) or high resistivity covering case (Fig.4), there are few difference between neglecting or considering displacement current when the frequency is lower than 2MHz. However, as the frequency increasing, the difference becomes obvious. So in layered model the displacement current effect should be considered when the frequency is high.

4.3 Simulation of time-domain Green function in whole space

For the time domain comparison, the whole space model was adopted. Setting the conductivity $\sigma=0.02S/m$, the relative dielectric constant $\epsilon_r=7$. And then calculating the Green function at the location which is 100m away from the unit source in time interval of $10^{-6.5} \sim 10^{-4}s$. The time domain Green function was calculated with equation (18) for considering the displacement current, while the equation (21) was adopted for neglecting case. The time responses were shown in figure 5.

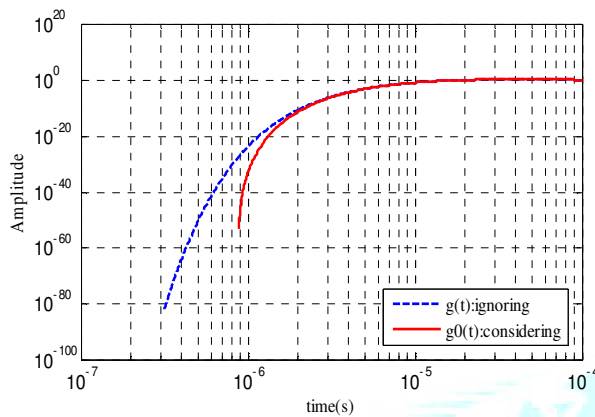


Fig.5 The curves of scalar time-domain Green functions

We can draw the following simulation conclusion from the above figure as: 1) there is no response at measuring point before $0.9\mu\text{s}$ when considering the displacement current because of the traveling time from source point to field point is about $0.88\mu\text{s}$; 2) In the time period of $0.89 \sim 4\mu\text{s}$, the response which considering the displacement current is smaller than the ignoring one (corresponding to quasi-static case); 3) as time increasing, the difference is gradually decreased. The numerical results showed that the relative error is 17.16% for $4\mu\text{s}$ and 10% for $4.67\mu\text{s}$. Therefore, the influence of displacement current effect is mainly reflected in the early time-domain response.

5. Conclusions

This paper analyzed the displacement current effect in the EM method for geophysical applications. The comparisons of considering and ignoring displacement current effect were carried out both in frequency and time domain. In frequency domain, the half-space and layered homogeneous models are adopted and the modified wave number calculation is implemented. And in time domain, the Green functions of whole space are compared.

The results show that the displacement current must be taken into account for high frequency (for example the frequency $f > 3\text{MHz}$), great resistivity and dielectric constant problems in frequency domain no matter the low resistivity or high resistivity covering. And for the time domain case, the displacement current effect is mainly reflected in the earlier time (for example $t < 5\mu\text{s}$ in simulation 3) response especially in the time near the range of traveling time.

Acknowledgments

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